Complex Numbers

Basic Concepts of Complex Numbers ■ Complex Solutions of Equations ■ Operations on Complex Numbers

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Identify the number as real, complex, or pure imaginary.

$2i$

The complex numbers are an extension of the real numbers. They include numbers of the form a + bi where a and b are real numbers.

Determine if $2i$ is a complex number.

$2i$ is a complex number because it can be expressed as $0 + 2i$, where 0 and 2 are real numbers.

The complex numbers include pure imaginary numbers of the form a + bi where a = 0 and b ≠ 0, as well as real numbers of the form a + bi where b = 0.

Choose the correct description(s) of $2i$: complex and pure imaginary.

Express in terms of i.

$$\sqrt{-196}$$

First, write – 196 as – 1 times 196.

$$\sqrt{-196}=\sqrt{-1(196)}$$

Next, write $\sqrt{-1(196)}$ as the product of two radicals.

$$\sqrt{-1(196)}=\sqrt{196}\cdot\sqrt{-1}$$

Finally, simplify each radical.

$$\sqrt{196}\sqrt{-1}=14i$$

Write the number as a product of a real number and i. Simplify all radical expressions.

$$\sqrt{-19}$$

First, write – 19 as – 1 times 19.

$\sqrt{-19} = \sqrt{(-1) \cdot 19}$

Next, write $\sqrt{(-1) \cdot 19}$ as the product of two radicals.

$\sqrt{(-1) \cdot 19} = \sqrt{-1} \cdot \sqrt{19}$

Simplify $\sqrt{-1}$.

$\sqrt{-1}\sqrt{19} = i\sqrt{19}$

Express in terms of i.

$\sqrt{-150}$

(Work out same as above.)

$5i\sqrt{6}$

Express in terms of i.

$-\sqrt{-50}$

$-\sqrt{-50} = -5i\sqrt{2}$

Solve the equation.

$x^2 = -25$

Take the square root of both sides.

$x = \pm\sqrt{-25}$

Rewrite the square root of the negative number.

$x = \pm i\sqrt{25}$

Simplify: $x = \pm 5i$

The solutions are $x = 5i, -5i$.

Solve the quadratic equation, and express all complex solutions in terms of i.

$x^2 = 4x - 20$

First, write the equation in standard form.

$$x^2 - 4x + 20 = 0$$

Use the quadratic formula to solve.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow a = 1,\ b = -4,\ c = 20$$

Substitute these values into the quadratic formula to get

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(20)}}{2(1)}$$

Simplify the radical expression.

$$x = \frac{4 \pm 8i}{2} \rightarrow 2 + 4i,\ 2 - 4i.$$

Multiply.

$$\sqrt{-8} \cdot \sqrt{-8}$$

Recall that $\sqrt{-8} = i\sqrt{8}$.

First, rewrite – 8 as – 1 · 8.

$$\sqrt{-8} \cdot \sqrt{-8} \quad = \quad \sqrt{-1(8)} \cdot \sqrt{-1(8)}$$

Split into several radicals.

$$\sqrt{-1(8)} \cdot \sqrt{-1(8)} \quad = \quad \sqrt{-1}\sqrt{8}\sqrt{-1}\sqrt{8}$$

Simply each radical, if possible.

$$\sqrt{-1}\sqrt{8}\sqrt{-1}\sqrt{8} \quad = \quad (i)(\sqrt{8})(i)(\sqrt{8})$$

Multiply.

$$(i)(\sqrt{8})(i)(\sqrt{8}) \quad = \quad i^2(\sqrt{8} \cdot \sqrt{8}) = -8$$

Divide.

$$\frac{\sqrt{-192}}{\sqrt{-64}}$$

$$\frac{\sqrt{-192}}{\sqrt{-64}} = \sqrt{3}$$

Divide.

$$\frac{\sqrt{-175}}{\sqrt{7}}$$

Notice that the expression in the numerator is imaginary.

$$\sqrt{-175}=i\sqrt{175}$$

Then simplify the radical.

$$i\sqrt{175}=5i\sqrt{7} \rightarrow \frac{5i\sqrt{7}}{\sqrt{7}}$$

Reduce the fraction to lowest terms by dividing out the common factor, $\sqrt{7}$.

$$\frac{5i\sqrt{7}}{\sqrt{7}} = 5i$$

Add and simplify.

$$(7+5i)+(2-4i)$$

$$(7+5i)+(2-4i) = 9 + i$$

Multiply.

$$(7+8i)(5+i)$$

$$(7+8i)(5+i) = 47i+27$$

Multiply.

$$(-6+9i)^2$$

$$(-6+9i)^2 = -108i-45$$

Multiply.

$(\sqrt{10}+i)(\sqrt{10}-i)$

$(\sqrt{10}+i)(\sqrt{10}-i) \quad = 11$

Simplify.

i^{17}

Since 17 is odd, rewrite 17 as 16 + 1 and simplify.

$i^{17}=i^{16+1}=i^{16}\cdot i$

Write 16 as 2(8).

$i^{16}\cdot i=i^{2(8)}\cdot i$

Write $i^{2(8)}$ as power of i^2 .

$i^{2(8)}\cdot i=(i^2)^8\cdot i$

Remember that $i^2=-1$ and simplify.

$(i^2)^8\cdot i=(-1)^8\cdot i$

Remember that a negative number raised to an even power is positive.

$(-1)^8\cdot i=1\cdot i \quad = i$

Simplify.

i^{34}

$i^{34}=-1$

Simplify.

i^{15}

$i^{15}=-i$

Find the power of i.

i^{-17}

Use the rule for negative exponents.

$$a^{-m}=\frac{1}{a^m}$$

$$i^{-17}=\frac{1}{a^{17}}$$

Because i^2 is defined to be – 1, higher powers of *i* can be found. Larger powers of *i* can be simplified by using the fact that $i^4=1$.

$$\frac{1}{i^{17}}=\frac{1}{i^4\cdot i^4\cdot i^4\cdot i^4\cdot i}$$

Since $i^4=1$, $\frac{1}{i^4\cdot i^4\cdot i^4\cdot i^4\cdot i} = \frac{1}{i}$

To simplify this quotient, multiply both the numerator and denominator by $-i$, the conjugate of *i*.

$$\frac{1}{i}=\frac{1(-i)}{i(-i)} \rightarrow \frac{-i}{-i^2} \rightarrow \frac{-i}{-(-1)} = -i$$

Divide.

$$\frac{8+4i}{8-4i}$$

Reduce.

$$\frac{2+i}{2-i}$$

Multiply by a form of 1 determined by the conjugate of the denominator.

$$\frac{2+i}{2-i}=\frac{2+i}{2-i}\cdot\frac{2+i}{2+i}=\frac{4+4i+i^2}{4-i^2}$$

Remember that $i^2=-1$ and simplify.

$$\frac{4+4i+i^2}{4-i^2}=\frac{4+4i+(-1)}{4-(-1)}$$

$$\frac{4+4i-1}{4+1}=\frac{3+4i}{5}$$

Write in the form a + bi.

$$\frac{3+4i}{5}=\frac{3}{5}+\frac{4}{5}i$$

Trigonometric (Polar) Form of Complex Numbers

The Complex Plane and Vector Representation ■ Trigonometric (Polar) Form ■ Converting Between Trigonometric and Polar Forms ■ An Application of Complex Numbers to Fractals

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Graph the complex number as a vector in the complex plane.

$$-4-7i$$

In the complex plane, the horizontal axis is called the real axis, and the vertical axis is called the imaginary axis.

The real part of the complex number is – 4.

The imaginary part of the complex number is – 7.

Graph the ordered pair $(-4,-7)$.

Draw an arrow from the origin to the point plotted.

Write the complex number in rectangular form.

18(cos 180° + i sin 180°)

Rewrite the equation in the form a + bi.

18(cos 180° + i sin 180°) = 18 cos 180° + (18 sin 180°)i

a = 18 cos 180°
b = 18 sin 180°

Simplify.

a = 18 cos 180°

a = (18)(– 1)

a = – 18

Simplify.

b = 18 sin 180°
b = 18 · 0

b = 0

18(cos 180° + i sin 180°) = – 18 + 0i = – 18

Write the complex number in rectangular form.

8(cos(30°) + i sin (30°))

$4\sqrt{3}+4i$

Write the complex number in rectangular form.

14 cis 315°

Rewrite the equation.

14 cis 315° = 14(cos 315° + i sin 315°)

14(cos 315° + i sin 315°) = 14 cos 315° + (14 sin 315°)i

a = 14 cos 315°
b = 14 sin 315°

Simplify.

a = 14 cos 315°

$a=14\cdot\frac{\sqrt{2}}{2}$

$a=7\sqrt{2}$

Simplify.

b = 14 sin 315°

b = $14\cdot-\frac{\sqrt{2}}{2}$

b = $-7\sqrt{2}$

$14\,\text{cis}\,315°=7\sqrt{2}-7\sqrt{2}\,\text{i}$

Find trigonometric notation.

5 – 5i

From the definition of trigonometric form of a complex number, you know that 5 – 5i = r(cos θ + i sin θ). Your goal is to find r and θ.

The definition says $r=\sqrt{a^2+b^2}$ for a complex number a + bi. So, for the complex number 5 – 5i, $r=\sqrt{(5)^2+(-5)^2}$.

In its reduced form, $\sqrt{(5)^2+(-5)^2}$ = $5\sqrt{2}$.

Use the value of r to find θ. For a complex number a + bi, a = r cos θ. For the complex number 5 – 5i, 5 = r cos θ and $r=5\sqrt{2}$, $5=5\sqrt{2}\cos\theta$.

Solve the equation for cos θ.

$$\cos\theta=\frac{5}{5\sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

Using the equation b = r sin θ and solving for sin θ, you get

$$\sin\theta=\frac{-5}{5\sqrt{2}}=\frac{-1}{\sqrt{2}}=-\frac{\sqrt{2}}{2} .$$

Placing the angle in the same quadrant as 5 – 5i, θ, written as a degree measure, is 315°.

$$5-5\mathrm{i}=5\sqrt{2}(\cos 315°+\mathrm{i}\sin 315°)$$

Write the complex number – 2 – 2*i* in trigonometric form r(cos θ + *i* sin θ), with θ in the interval [0°,360°].

– 5 – 5*i* = Find r by using the equation $r=\sqrt{x^2+y^2}$ for the number x y*i*.

$$r=\sqrt{(-2)^2+(-2)^2} \rightarrow 2\sqrt{2}$$

Use the value of r to find θ. For a complex number x + y*i*, x = r cos θ.

$$-2=2\sqrt{2}\cos\theta .$$

Solve the equation for cos θ.

$$\cos\theta=\frac{-2}{2\sqrt{2}} \rightarrow = \frac{-1}{\sqrt{2}}$$

Solve for θ.

$$\theta = \cos^{-1}(\frac{-1}{\sqrt{2}})$$

Placing the angle in the same quadrant as – 5 – 5i, θ, written in degree measure, is 225°.

$$-2-2i = 2\sqrt{2}(\cos 225° + i \sin 225°)$$

Find the trigonometric form. $0 \leq \theta < 360$

3

From the definition of trigonometric form of a complex number, you know that 3 = r(cos θ + i sin θ). Your goal is to find r and θ.

The definition says $r = \sqrt{a^2 + b^2}$ for a complex number a + bi. So, for the complex number 3, $r = \sqrt{(3)^2 + (0)^2}$.

In its reduced form, $\sqrt{(3)^2 + (0)^2} = 3$.

Use the value of r to find θ. For a complex number a + bi, a = r cos θ. For the complex number 3, 3 = r cos θ and r = 3, so 3 = 3 cos θ. Solve the equation for cos θ.

$$\cos\theta = \frac{3}{3} = 1$$

$$\theta = \cos^{-1}(1)$$

θ, written as a degree measure, is 0°.

3 = 3 (cos 0° + i sin 0°)

Write the complex number 6 (cos 98° + i sin 98°) in the form a + bi.

To solve this problem, first find the cosine of 98° and the sine of 98°.

The cosine of 98° is $-.139173$, rounded to six decimal places.

The sine of 98° is $.990268$, rounded to six decimal places.

$$6(\cos 98° + i \sin 98°) = 6(-0.139173 + i\, 0.990268) \quad \rightarrow$$

$$6 \cdot -0.139173 + i\, 6 \cdot 0.990268$$

So, the answer in a + bi form is – 0.8 + 5.9 i.

Convert the rectangular form $9i$ into trigonometric form.

To find r, use the formula $r=\sqrt{x^2+y^2}$.

In this case, x = 0 and y = 9.

$$r=\sqrt{(0)^2+(9)^2} \rightarrow r=\sqrt{0+81} \rightarrow r=9$$

Sketch the graph of $9i$. Note that because x is 0, θ is a quadrantal angle.

θ = 90°

The trigonometric form of the complex number x + yi is r(cos θ + i sin θ). Substitute the values of r and θ into this form to find the trigonometric form of $9i$.

$9i = 9\ (\cos 90° + i \sin 90°)$

Convert the rectangular form – 3 + 5i into trigonometric form.

$$r=\sqrt{x^2+y^2}$$

In this case, x = – 3 and y = 5.

$$r=\sqrt{(-3)^2+(5)^2} \rightarrow r=\sqrt{9+25} \rightarrow r\sqrt{34} \rightarrow$$

r = 5.8, rounded to nearest tenth.

To find the degree measure of the and θ, use the following formula.

$$\theta=\tan^{-1}\left(\frac{y}{x}\right) \text{ if } x>0 \text{ or } \theta=180+\tan^{-1}\left(\frac{y}{x}\right) \text{ if } x<0.$$

Since x < 0, use the second formula.

$$\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{5}{-3}\right)$$

≈ – 59.0363° → θ ≈ – 59.0363°

Add 180 to get positive angle.

θ ≈ 121.0°, rounded to nearest tenth.

$-3 + 5i = 5.8(\cos 121.0° + i \sin 121.0°)$

The Product and Quotient Theorems

Products of Complex Numbers in Trigonometric Form ■

Quotients of Complex Numbers in Trigonometric Form

Multiply the two complex numbers.

3(cos 146° + i sin 146°) · 7(cos 102° + i sin 102°)

3(cos 146° + i sin 146°) · 7(cos 102° + i sin 102°) =

21 (cos 248° + i sin 248°)

Convert your answer to a + bi form.

21(cos 248° + i sin 248°) = 21 (– 0.3746066 +i(– 0.92718385))
= – 7.8667386 + (– 19.47086085)i

Rounding each number to the nearest tenth gives the following.

3(cos 146° + i sin 146°) · 7(cos 102° + i sin 102°)
= – 7.9 – 19.5i

Find the following product, and write the product in rectangular form, using exact values.

[2(cos 60° + *i* sin 60°)][5(cos 180° + *i* sin 180°)]

To multiply two complex numbers, use the fact that if they are expressed in polar form $r_1(\cos\theta+i\sin\theta_1)$ and $r_2(\cos\theta_2+\sin\theta_2)$, there product is $r_1r_2(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2))$.

Multiply these two complex numbers.

[2(cos 60° + i sin 60°)][5(cos 180° + i sin 180°)]

= (2 · 5)[cos (60° + 180°) + i sin (60° + 180°)]

= 10[cos(240°) + i sin (240°)]

Determine the exact values of cos 240° and sin 240°. Replace these into the expression and simplify the result.

$$= \quad 10\left(-\frac{1}{2}+i-\frac{\sqrt{3}}{2}\right) \rightarrow 10\left(-\frac{1}{2}\right)+i(10)\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \quad -5-5\sqrt{3}i$$

Find the following product, and write the result in rectangular form using exact values.

(2 cis 45°)(5 cis 105°)

Rewrite.

(2 cis 45°)(5 cis 105°) = $[2(\cos 45° + i\sin 45°)][5(\cos 105° + i\sin 105°)]$

To multiply two complex numbers, use the fact that if they are expressed in polar form $r_1(\cos\theta + i\sin\theta_1)$ and $r_2(\cos\theta_2 + \sin\theta_2)$, there product is $r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$.

Multiply these two complex numbers.

[2(cos 45° + i sin 45°)][5(cos 105° + i sin 105°)]

= (2 · 5)[cos(45° + 105°) + i sin (45° + 105°)]

=10[cos(150°) + i sin (150°)]

Notice that you know the exact values of cos 150° and sin 150°. Replace these into the expression and simplify the result.

(2 cis 45°)(5 cis 105°) = $10(-\frac{\sqrt{3}}{2} + (\frac{1}{2})i)$ →

$10(-\frac{\sqrt{3}}{2}) + (10)(\frac{1}{2})i \quad = \quad -5\sqrt{3} + 5i$

Find the quotient and write it in rectangular form using exact values.

$$\frac{20(\cos 120° + i\sin 120°)}{5(\cos 180° + i\sin 180°)}$$

To simplify the quotient, first use the fact that

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)) .$$

$$\frac{20(\cos 120° + i\sin 120°)}{5(\cos 180° + i\sin 180°)} = 4(\cos -60° + i\sin -60°)$$

Next, write this product in rectangular form by evaluating the cosine and sine functions.

cos – 60° = $\frac{1}{2}$

sin – 60° = $-\frac{\sqrt{3}}{2}$

Expand the final product.

$$\frac{20(\cos 120° + i\sin 120°)}{5(\cos 180° + i\sin 180°)} = 2 - 2\sqrt{3}i$$

Use a calculator to perform the indicated operations.

$$\frac{73(\cos 109 + i\sin 109)}{36.5(\cos 69 + i\sin 69)}$$

= 1.53 + 1.29 *i* (rounded to nearest hundredth)

Use a calculator to perform the indicated operation. Give the answer in rectangular form.

$$\left(2\, cis \frac{6\pi}{7}\right)^2$$

Use the fact that $x^2 = x \cdot x$ to rewrite the problem.

$$\left(2\, cis \frac{6\pi}{7}\right)^2 = \left(2\, cis \frac{6\pi}{7}\right)\left(2\, cis \frac{6\pi}{7}\right)$$

Use the product rule.

$$\left(2\, cis \frac{6\pi}{7}\right)\left(2\, cis \frac{6\pi}{7}\right) = 4\, cis \frac{12\pi}{7}$$

Remember cis θ = cos θ + i sin θ. Rewrite the right side of the equation.

$$4\, cis \frac{12\pi}{7} = 4\left(\cos\frac{12\pi}{7} + i\sin\frac{12\pi}{7}\right)$$

Using a calculator, find the values of $\cos\frac{12\pi}{7}$ and $\sin\frac{12\pi}{7}$.

$$4\, cis \frac{12\pi}{7} = 4(0.623490 - 0.781831i)$$

Multiply through by 4.

$$4(0.623490 - 0.781831i) = 2.4940 - 3.1273i$$

The alternating current in an electric inductor is $I = \frac{E}{Z}$, where E is voltage and $Z = R + X_L i$ is impedance. If

$E = 5(\cos 50° + i \sin 50°)$, R = 7, and $X_L = 4$, find the current.

To find the quotient, it is convenient to write all complex numbers in trigonometric form. E is already in trigonometric form, though you can write it more conveniently.

E = 5 cis 50°

Convert Z into trigonometric form.

To convert from rectangular form x + yi to trigonometric form r cis θ, remember $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$. Use these equations to solve for r and θ.

Z = 8.0623 cis 29.7449° (rounded to four decimal places)

Substitute in the values for E and Z, and then apply the quotient rule.

$$I = \frac{E}{Z} = \frac{5\,cis\,50°}{8.0623\,cis\,29.7449°}$$

The quote of two complex numbers in trigonometric form may be simplified by

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)) .$$

= 0.6202 cis 20.2551° (rounded to four decimal places)

Convert back to rectangular form by expanding.

Remember that cis θ means cos θ + i sin θ. Use a calculator to evaluate the sine and cosine of 28.1986°.

0.6202 cis 20.2551° = 0.58 + 0.21 *i*

De Moivre’s Theorem; Powers and Roots of Complex Numbers

Powers of Complex Numbers (De Moivre’s Theorem) ■ Roots of Complex Numbers

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Find the following power. Write the answer in rectangular form.

$$[5(\cos 30° + i\sin 30°)]^2$$

To find the powers of a complex number, use De Moivre's theorem.

$$[5(\cos 30° + i\sin 30°)]^2 = 5^2(\cos 2\cdot 30° + i\sin 2\cdot 30°)$$

$= 25(\cos 60° + i \sin 60°)$

Find the exact values of cos 60° and sin 60° and simplify.

$= \quad 25\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$

In a + bi form: $\frac{25}{2}+\frac{25\sqrt{3}}{2}i$

Raise the number to the given power and write the answer in rectangular form.

$[\sqrt{10}(cis\,120°)]^4$

Use De Moivre's theorem.

$[r(\cos\theta+i\sin\theta)]^n=r^n(\cos n\theta+i\sin n\theta)$

The modulus $\sqrt{10}$, raised to the power of 4, is 100.

The argument 120°, multiplied by 4, is 480°.

$[\sqrt{10}(cis\,120°)]^4 = 100(\cos 480° + i \sin 480°)$

Use the distributive property.

$100(\cos 480° + i \sin 480°) = 100 \cdot \cos 480° + 100 \cdot i \sin 480°$

The product $100 \cdot \cos 480°$ is equal to – 50.

The product of $100 \cdot \sin 480°$ is equal to $50\sqrt{3}i$.

Add.

$[\sqrt{10}(cis\,120°)]^4 \quad = \quad -50+50i\sqrt{3}$

Find the given power.

$(4+4i)^7$

TI-83: $(4+4i)^7 = 131072 - 131072\,i.$

Find the cube roots of $125(\cos(0°) + i \sin(0°))$.

Find the cube roots of 125(cos(0°) + i sin (0°)).

The number of cube roots of 125(cos(0°) + i sin (0°)) is 3.

The roots of a complex number with modulus r all have the same modulus, $\frac{1}{r^n}$. In this case, r = 125.

$$\frac{1}{r^n} = (125)^{\frac{1}{3}} = 5$$

The n nth roots of a complex number are distinguished by n different arguments. Find these arguments by using the following formula.

$$\alpha = \frac{\theta + 2k\pi}{n} = \frac{\theta + k \cdot 360°}{n} \quad \text{for} \quad k = 0, 1, 2, \ldots, n-1 \ .$$

In this case n = 3, θ = 0°, and k = 0, 1, and 2.

When k = 0, α = $\frac{0° + (0)(360°)}{3}$ = 0°.

That gives the complex root 5(cos 0° + i sin 0°).

When k = 1, α = $\frac{0° + (1)(360°)}{3}$ = 120°.

This gives the complex root 5(cos 120° + i sin 120°).

When k = 2, α = $\frac{0° + (2)(360°)}{3}$ = 240°.

This gives the complex root 5(cos 240° + i sin 240°).

The complex roots are:
5(cos 0° + i sin 0°),
5(cos 120° + i sin 120°), and
5(cos 240° + i sin 240°).

Find the cube roots of 27 cis 90°.

Find the cube roots of 27(cos 90° + i sin 90°). The number of cube roots(cos 90° + i sin 90°) is 3.

The roots of a complex number with modulus r all have the same modulus, $r^{\frac{1}{n}}$. In this case, r = 27.

$$r^{\frac{1}{n}} = (27)^{\frac{1}{3}} = 3$$

The n nth roots of a complex number are distinguished by n different arguments. Find these arguments by using the following formula.

$$\alpha=\frac{\theta+2k\pi}{n}=\frac{\theta+k\cdot 360°}{n} \quad \text{for} \quad k=0,1,2,\ldots,n-1 \quad .$$

In this case n = 3, θ = 90°, and k = 0, 1, and 2.

When k = 0, $\alpha=\dfrac{90°+(0)(360°)}{3}=30°$.

This gives the complex root 3(cos 30° + i sin 30°).

When k = 0, $\alpha=\dfrac{90°+(1)(360°)}{3}=150°$.

This gives the complex root 3(cos 150° + i sin 150°).

When k = 0, $\alpha=\dfrac{90°+(2)(360°)}{3}=270°$.

This gives the complex root 3(cos 270° + i sin 270°).

The complex roots are:
3(cos 30° + i sin 30°),
3(cos 150° + i sin 150°), and
3(cos 270° + i sin 270°).

Find the cube roots of the following complex number. Express your answer in 'cis' form. Then plot the cube roots.

– 8

To begin, convert – 8 into the polar form of the same number.

To express – 8 in polar form, begin by sketching the graph of – 8 in the complex plane, then find the vector with direction θ and magnitude r corresponding to the number in rectangular form.

Remember that $r=\sqrt{x^2+y^2}$ and $\tan\theta=\dfrac{y}{x}$.

In the polar form, – 8 = 8 cis 180.

Next, find the cube roots. These are the numbers of the form a + bi, where $(a+bi)^3$ is equal to – 8. Suppose this complex number is r(cos α + i sin α). Then you want the third power to be equal to 8(cos 180° + i sin 180°).

$$r^3(\cos 3\alpha+i\sin 3\alpha)=8(\cos 180°+i\sin 180°)$$

Satisfy part of this equation by letter $r^3=8$. Solve this for r.

$r^3=8$ → r = 2

Next, make nα an angle coterminal with 180°. Therefore, you must have 3 α = 180° + 360° · k, where k is any nonnegative integer less than n.

α = 60° + 120° · k

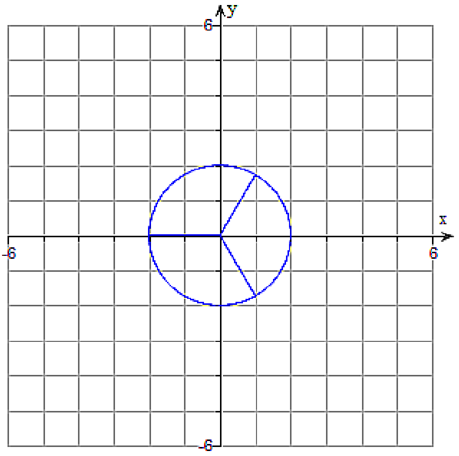
Let k take on integer values, starting with 0. If k = 0, then α = 60°.

If k = 1, then α = 180°.

If k = 2, α = 300°. Notice that if k = 3, α = 60° + 360°, which will be coterminal with 60°, so you would just be repeating solutions already found if higher values of k were used.

The solutions will be 2 cis 60°, 2 cis 180°, and 2 cis 300°.

The three solutions can be graphed as shown below.



Find the cube roots of the following complex number. Then plot the cube roots.

$-7\sqrt{3}+7i$

To begin, convert $-7\sqrt{3}+7i$ into the polar form of the same number.

To express $-7\sqrt{3}+7i$ in polar form, begin by sketching the graph of $-7\sqrt{3}+7i$ in the complex plane, then find the vector with direction θ and magnitude r corresponding to the number in rectangular form. Remember that $r=\sqrt{x^2+y^2}$ and $\tan\theta=\frac{y}{x}$.

In polar form, $-7\sqrt{3}+7i$ = 14 cis 150.

Next, find the cube roots. These are the numbers of the form a + bi, where $(a+bi)^3$ is equal to $-7\sqrt{3}+7i$. Suppose this complex number is r(cos α + i sin α). Then you want the third power to be equal to 14 cis 150°.

r^3 (cos 3 α + i sin 3 α) = 14 (cos 150° + i sin 150°)

Satisfy part of this equation by letting $r^3 = 14$. Solve this for r.

$r^3 = 14 \rightarrow \sqrt[3]{14}$

Next, make n α an angle coterminal with 150°. Therefore, you must have 3 α = 150° + 360° · k, where k is any nonnegative integer less than n.

α = 50° + 120° · k

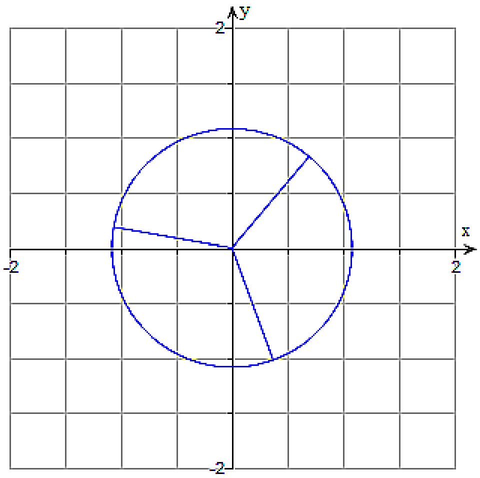
Now let k take on integer values, starting with 0.

If k = 0, then alpha = 50°.

If k =1, then α = 170°.

If k = 2, then α = 290°. Notice the if k = 3, α = 50° + 360°, which will be coterminal with 50°, so you would just be repeating solutions already found if higher values of k were used.

The solutions will be $\sqrt[3]{14}\,cis\,50°$, $\sqrt[3]{14}\,cis\,170°$, and $\sqrt[3]{14}\,cis\,290°$.



Find all solutions to the equation.

$x^3 + 512 = 0$

Since $x^3 + 512 = 0$, $x^3 = -512$. 'Solving the equation' is equivalent to finding the cube roots of – 512. Begin by writing –512 in trigonometric form.

In trigonometric form, the modulus is 512.

Find the smallest nonnegative value θ for which – 512 = 512 cos θ.

The argument, written as a degree measure, is 180°.

– 512 = 512(cos 180° + i sin 180°)

The number of cube roots of 512(cos 180° + i sin 180°) is 3.

The roots of a complex number with modulus r all have the same modulus $r^{\frac{1}{n}}$. In this case r = 512.

$r^{\frac{1}{n}} = 512^{\frac{1}{3}} = 8$

Then n nth roots of a complex number are distinguished by n different arguments. You can find these arguments by using the following formula.

$\alpha=\frac{\theta+360\cdot k}{n}$ for $k=0,1,2,\ldots,n-1$.

In this case, n = 3, θ = 180°, and k = 0, 1, and 2.

When k = 0, $\alpha=\frac{180°+(360°)(0)}{3}$ = 60°.

This gives you the complex root 8(cos 60° + i sin 60°).

When k = 0, $\alpha=\frac{180°+(360°)(1)}{3}$ = 180°.

This gives you the complex root 8(cos 180° + i sin 180°).

When k = 2, $\alpha=\frac{180°+(2)(360°)}{3}$ = 300°.

This gives you the complex root 8(cos 300° + i sin 300°).

The solutions to $x^3+512=0$ are:
8(cos 60° + i sin 60°),
8(cos 180° + i sin 180°), and
8(cos 300° + i sin 300°).

Find all solutions to the equation $x^3+64=0$.

Since $x^3+64=0$, $x^3=-64$. "Solving the equation" is equivalent to finding the cube roots of – 64. Begin by writing – 64 in trigonometric form.

Find the modulus using $\sqrt{(0)^2+(-64)^2}$ and simplify the radical.

In trigonometric form, the modulus is 64.

You want to find the smallest nonnegative value θ for which – 64 =

$64\cos\theta$.

The argument, written as a degree measure, is 180°.

$-64 = 64(\cos 180° + i \sin 180°)$.

The number of cube roots of $64(\cos 180° + i \sin 180°)$ is 3.

The roots of a complex number with modulus r all have the same modulus $r^{\frac{1}{n}}$.

In this case r = 64, so $r^{\frac{1}{n}} = 64^{\frac{1}{3}} = 4$.

The n nth roots of a complex number are distinguished by n different arguments. You can find these arguments by using the formula

$\alpha = \frac{\theta + 2k\pi}{n} = \frac{\theta + k \cdot 360°}{n}$ for k = 0,1,2,...,n – 1. In this case, n = 3, θ = 180°, and k = 0, 1, and 2.

When k = 0, $\alpha = \frac{180° + (0)(360°)}{3} = 60°$.

This gives the complex root $4(\cos 60° + i \sin 60°)$.

$= 4\left(\frac{1}{2} + i \cdot \frac{\sqrt{3}}{2}\right) = 2 + 2i\sqrt{3}$

When k = 1, $\alpha = \frac{180° + (1)(360°)}{3} = 180°$.

$= 4(-1 + i(0)) = -4$

When k = 2, $\alpha = \frac{180° + (2)(360°)}{3} = 300°$.

This gives the complex root $4(\cos 300° + i \sin 300°)$.

$= 4\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = 2 - 2i\sqrt{3}$

Find all complex solutions.

$x^4 + 10 = 0$

To solve this equation, first isolate the power on one side of the equals sign, and take the fourth root.

$x^4 + 10 = 0 \rightarrow x^4 = -10$

To find the fourth root, first write – 10 in polar form.

To express – 10 in polar form, begin by sketching the graph of – 10 in the complex plane, then find the vector with direction θ and magnitude r that corresponds to the number in rectangular form.

Remember that $r=\sqrt{x^2+y^2}$ and $\tan\theta=\frac{y}{x}$.

In polar form, – 10 = 10 cis 180

Now, take the fourth roots. These are the numbers of the form a + bi, where $(a+bi)^4$ is equal to 10 cis 180. Suppose this copmlex number is r9cos α + i sin α). Then you want the fourth power to be equal to – 10, or 10 cis 180°.

$$r^4(\cos 4\alpha+i\sin 4\alpha)=10(\cos 180°+i\sin 180°)$$

Satisfy part of this equation by letting $r^4=10$. Solve this for r.

$$r^4=10 \rightarrow r=\sqrt[4]{10}$$

Next, make n α an angle coterminal with 180°. Therefore, you must have 4 α = 180° + 360° · k, where k is any nonnegative integer less than n.

α = 45 + 90k

Let k take on integer values, starting with 0.

If k = 0, then α = 45°.

If k = 1, then α = 135°.

If k = 2, then α = 225°, and if k = 3, then α = 315°.

Notice that if k = 4, then α = 180° + 360°, which will be coterminal with 180°, so you would just be repeating solutions already found if higher values of k were used.

The solutions will be $\sqrt[4]{10}$ cis 45°, $\sqrt[4]{10}$ cis 135°, $\sqrt[4]{10}$ 225°, and $\sqrt[4]{10}$ cis 315°.

Find and graph the cube roots of 216i.

Begin by writing 216i in trigonometric form.

The modulus is $\sqrt{(0)^2+(216)^2}$. Simplify the radical.

In trigonometric form, the modulus is 216.

Find the smallest nonnegative value θ for which 216 = 216 · sin θ.

The argument, written as a nonnegative degree measure, is 90°.

$216i = 216(\cos 90° + i \sin 90°)$

The number of cube roots of $216(\cos 90° + i \sin 90°)$ is 3.

The roots of a complex number with modulus r all have the same modulus $r^{\frac{1}{n}}$.

In this case r = 216.

$$r^{\frac{1}{n}} = 216^{\frac{1}{3}} = 6$$

The n nth roots of a complex number are distinguished by n different arguments. Find these arguments by using the following formula.

α = {θ + 360° · k} over n for k = 0, 1, 2,...,n – 1.

In this case, n = 3, θ = 90°, k = 0, 1, and 2.

When k = 0,

$$\alpha = \frac{90° + (360°)(0)}{3} = 30° .$$

This gives the complex root $6(\cos 30° + i \sin 30°)$.

When k = 1,

$$\alpha = \frac{90° + (360°)(1)}{3} = 150° .$$

This gives the complex root $6(\cos 150° + i \sin 150°)$.

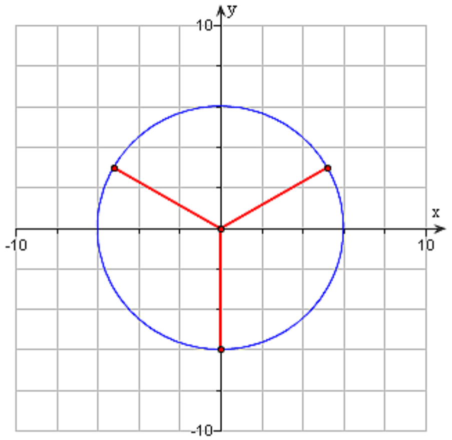
When k = 2, $\alpha = \frac{90° + (360°)(2)}{3} = 270°$.

This gives the complex root $6(\cos 270° + i \sin 270°)$.

These three roots can be written in rectangular form and plotted.

$P_1 = (5.196, 3.000)$,
$P_2 = (-5.196, 3.000)$, and
$P_3 = (0.000, -6.000)$

If these are the correct roots, the points will lie on a circle with center at (0,0) and radius 6 (the modulus of the roots). The three points will divide the circumference of the circle in thirds.



Find and graph the fourth roots of 256.

Listed below are 8 choices lettered from a – h. Choose the correct solution (s) by typing the corresponding letters separated by commas.

a) 4(cos 0° + i sin 0°)
b) 4(cos 90° + i sin 90°)
c) 4(cos 180° + i sin 180°)
d) 4(cos 270° + i sin 270°)
e) 4(cos 45° + i sin 45°)
f) 4(cos 135° + i sin 135°)
g) 4(cos 225° + i sin 225°)
h) 4(cos 315° + i sin 315°)

Begin by writing 256 in trigonometric form. In trigonometric form, the modulus is $\sqrt{(256)^2+(0)^2}$ = 256.

To find the argument, find the smallest nonnegative value of θ for which 256 = 256 · cos θ.

The argument, written as a nonnegative degree measure, is 0°.

256 = 256(cos 0° + i sin 0°)

There are 4 fourth roots of 256(cos 0° + i sin 0°).

The roots of a complex number with modulus r all have the same modulus $r^{\frac{1}{n}}$. In this case r = 256.

$$r^{\frac{1}{n}}=256^{\frac{1}{4}} = 4$$

The n nth roots of a complex number are distinguished by n different arguments. Find these arguments by using the following formula.

$$\alpha=\frac{\theta+360°\cdot k}{n} \quad \text{for } k = 0,1,2,...n-1.$$

In this case, n = 4, $\theta = 0°$, k = 0, 1, 2 and 3.

When = 0,

$$\alpha = \frac{0° + (360°)(0)}{4} = 0°.$$

This gives the complex root 4(cos 0° + i sin 0°).

When k = 1,

$$\alpha = \frac{0° + (360°)(1)}{4} = 90°.$$

This gives the complex root 4(cos 90° + i sin 90°).

Note that α = 180° and 270° for k = 2 and 3 respectively. The final two complex roots are 4(cos 180° + i sin 180°) and 4(cos 270° + i sin 270°).

The graph of the solution set shows all solutions are spaced 90° apart.