Calculate the area under $f(x) = 4 - x^2$ [1,2].

$$\int_a^b f(x)\,dx = \lim_{x\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{2-1}{n} = \mathbf{\frac{1}{n}}$$

$$x_i^* = a + (\Delta x)i = 1 + \left(\frac{1}{n}\right)i = \mathbf{1 + \frac{i}{n}}$$

$$f(x_i^*) = 4 - \left(1 + \frac{i}{n}\right)^2 = 4 - \left(1 + \frac{2i}{n} + \frac{i^2}{n^2}\right) = 4 - 1 - \frac{2i}{n} - \frac{i^2}{n^2} = \mathbf{3 - \frac{2i}{n} - \frac{i^2}{n^2}}$$

$$\int_1^2 f(4 - x^2)\,dx = \lim_{x\to\infty} \sum_{i=1}^{n} \left(\mathbf{3 - \frac{2i}{n} - \frac{i^2}{n^2}}\right) \cdot \left(\frac{1}{n}\right)$$

$$\lim_{n\to\infty} \frac{1}{n}\left(\sum_{i=1}^{n} 3 - \frac{2}{n}\sum_{i=1}^{n} i - \frac{1}{n^2}\sum_{i=1}^{n} i^2\right)$$

Use summation formulas.

$$\lim_{n\to\infty} \frac{1}{n}\left(3n - \frac{2}{n}\left(\frac{n(n+1)}{2}\right) - \frac{1}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)\right)$$

Distribute $\frac{1}{n}$ back inside.

$$\lim_{n\to\infty} \left[\frac{3n}{n} - \frac{2}{n^2}\left(\frac{n^2+n}{2}\right) - \frac{1}{n^3}\left(\frac{2n^3+\cdots}{6}\right)\right]$$

Note that in $\left(\frac{2n^3+\cdots}{6}\right)$, the most important thing is the power of the numerator. We can save time by just realizing that there is an n^3 in the numerator.

$$\lim_{n\to\infty} \left(3 - \frac{2n^2}{2n^2} - \frac{2n}{2n^2} - \frac{2n^3+\cdots}{6n^3}\right)$$

Find the limits. Recall that the limit of a constant is the constant itself. Recall also that when the power of numerator and denominator are the same, you can simply use the coefficient of the numerator and denominator.

$$= \left(3 - 1 - \frac{1}{n} - \frac{1}{3}\right) = 3 - 1 - 0 - \frac{1}{3} = \frac{6}{3} - \frac{1}{3} = \frac{5}{3}$$

Visual representation of the integral: