

35. Use the guidelines of curve sketching to sketch the curve.

$$y = \frac{1}{2}x - \sin x, 0 < x < 3\pi$$

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**Domain:** Given.

**Intercepts:** We cannot set  $x=0$  because the domain excludes  $x$ .

$x$ -int.: This is difficult to find at this stage. So, we will wait to see if it is needed.

**Symmetry:** This function would be odd if we could include negative  $x$  values. But, the domain excludes such values.

**Asymptotes:** There are no asymptotes at the endpoints because our function would be defined. It would be continuous if we were to go beyond the points 0 and  $3\pi$ . So, there are no vertical asymptotes. There are no horizontal asymptotes because we cannot let a  $x$  go to positive or negative infinity due to the limited domain.

**Decreasing/Increasing Intervals:**

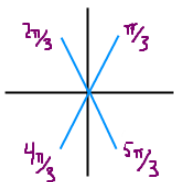
$$f'(x) = \frac{1}{2} - \cos(x)$$

{ Find points where the function equals zero to find critical points. }

$$f'(x) = 0 = \frac{1}{2} - \cos(x)$$

{ Find where  $\cos$  is  $1/2$  between  $0$  and  $3\pi$ . }

$\arccos$  of  $\frac{1}{2} = \pi/3$



{  $\cos$  is positive in the first and fourth quadrants. }

Note that the interval goes to  $3\pi$ .

$$\cos x = 1/2 @ \pi/3, 5\pi/3, 7\pi/3$$

Int.	Test Value	Sign	Inc./Dec. of $f$
$(0, \pi/3)$	$f'(5)$	-	↓
$(\pi/3, 5\pi/3)$	$f'(4)$	+	↑
$(5\pi/3, 7\pi/3)$	$f'(6)$	-	↓
$(7\pi/3, 3\pi)$	$f'(8)$	+	↑

**Local Min./Max.:**

Loc. min.:

$$\left(\frac{\pi}{3}, f\left(\frac{\pi}{3}\right)\right) = \left(\frac{\pi}{3}, -0.34\right)$$

$$\left(\frac{7\pi}{3}, f\left(\frac{7\pi}{3}\right)\right) = \left(\frac{7\pi}{3}, 2.80\right)$$

Loc. max.:

$$\left(\frac{5\pi}{3}, f\left(\frac{5\pi}{3}\right)\right) = \left(\frac{5\pi}{3}, 3.48\right)$$

## Concavity:

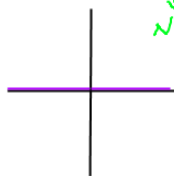
$$f''(x) = 0 + \sin(x) \\ = \sin(x)$$

$$f''(x) = 0 = \sin(x)$$

$$\arcsin(0) = 0$$

$$\sin(x) = 0 @ \cancel{0}, \pi, 2\pi$$

↓  
Not in Domain.



Int	Test Value	Sign of $f''$	Concavity
$(0, \pi)$	$f''(2)$	+	∪
$(\pi, 2\pi)$	$f''(4)$	-	∩
$(2\pi, 3\pi)$	$f''(7)$	+	∪

## Inflection Points:

$$(\pi, f(\pi)) = \left(\pi, \frac{\pi}{2}\right)$$

$$(2\pi, f(2\pi)) = (2\pi, \pi)$$

# Graph:

