23. Use the guidelines of curve sketching to sketch the curve.

$$y = \frac{x}{\sqrt{x^2+1}}$$

Domain: $(-\infty, \infty)$

Intercepts:

$f(0) = 0$

$0 = \frac{x}{\sqrt{x^2+1}} = 0$

Symmetry:

Even?

$f(2) = f(-2)$?

$\downarrow \qquad \downarrow$

$.89 \qquad -.89 \quad \neq$

Odd?

$-f(-2) = f(-2)$?

$\downarrow \qquad \downarrow$

$-.89 \qquad -.89 \quad =$

Asymptotes:

No vertical asymptotes as there is no place where function is undefined per domain.

HA:

$$\lim_{x \to \infty} \frac{x}{\sqrt{x^2+1}}$$

$$f(99999) = 1$$

$$\lim_{x \to \infty} \frac{x}{\sqrt{x^2+1}}$$

$$f(-99999) = -1$$

Decreasing/Increasing Intervals:

$$f'(x) = \frac{(x)'(x^2+1)^{1/2} - (x)\left[(x^2+1)^{1/2}\right]'}{\left(\sqrt{x^2+1}\right)^2}$$

$$= \frac{(x^2+1)^{1/2} - (x)\left[\cancel{\tfrac{1}{2}}(x^2+1)^{-1/2}\right] \cdot \cancel{2}x}{x^2+1}$$

{Take out an $(x^2+1)^{-1/2}$}

$$= \frac{(x^2+1)^{-1/2} \cdot (x^2+1-x^2)}{x^2+1}$$

{Note that $(x^2+1)^{-1/2}$ factored out of $(x^2+1)^{-1/2}$

is the same as $\dfrac{(x^2+1)^{1/2}}{(x^2+1)^{-1/2}}$

$$= (x^2+1)^{1/2} \cdot (x^2+1)^{1/2}$$

$$= \sqrt{x^2+1} \cdot \sqrt{x^2+1}$$

$$= x^2+1$$}

$$= \frac{(x^2+1)^{-1/2}\cdot(1)}{x^2+1} = \frac{1}{(x^2+1)^{1/2}(x^2+1)}$$

$\{$ Add exponents to get 3/2. $\}$

$$= \frac{1}{(x^2+1)^{3/2}}$$

The top and bottom will always be positive here. A positive multiplied by a positive is positive. So, the function should always be increasing.

$$f'(x) > 0 \text{ on } (-\infty, \infty)$$

$$f(x) \text{ is } \nearrow \text{ on } (-\infty, \infty)$$

Local Min./Max.:
There is no local min. or max. because, as we see in the previous step, there is no place where the derivative changes signs. There are, therefore, no places where the function changes from increasing to decreasing.

Concavity:

$$f'(x) = (x^2+1)^{-3/2}$$

$$f''(x) = -\tfrac{3}{2}(x^2+1)^{-5/2}\cdot 2x$$

$$= \frac{-6x}{2(x^2+1)^{5/2}}$$

$$= \frac{-3x}{(x^2+1)^{5/2}}$$

The second interval is 0 when $x = 0$. So, if we look at the intervals $(-\infty, 0)$ and $(0, \infty)$, we can see whether or not these are points of change in concavity.

		$-3x$	$(x^2+1)^{5/2}$	Prod	f
$(-\infty, 0)$	$f(-999)$	$+$	$+$	$+$	$\cup$
$(0, \infty)$	$f(999)$	$-$	$+$	$-$	$\cap$

Inflection Points:
The graph changes concavity at (0, 0).

Graph: