3. Use the guidelines of this section to sketch the curve.

$y = 2 - 15x + 9x^2 - x^3$

Equation Rewritten:

$y = -x^3 + 9x^2 - 15x + 2$

Domain: $(-\infty, \infty)$

Intercepts:
y-int.: $f(0) = 2$

You can see below that solving $f(x) = 0$ is somewhat messy. We may be able to skip this step and see if we need it later.

$f(x) = 0$

$0 = -x^3 + 9x^2 - 15x + 2$

$2 = x^3 - 9x^2 + 15x$

$2 = x(x^2 - 9x + 15)$

$\frac{9 \pm \sqrt{(-9)^2 - 4(1)(15)}}{2(1)}$

$\frac{9 \pm \sqrt{81 - 60}}{2} = \frac{9 \pm \sqrt{21}}{2}$

$2 = 6.79x \Rightarrow x \approx .295$

$2 = 1.10x \Rightarrow x \approx 1.82$

Symmetry:
We can see that there is no symmetry because the

function includes odd-powered terms and an even-powered term.

Asymptotes:
There will be no vertical asymptotes because there is no place where the function is undefined.

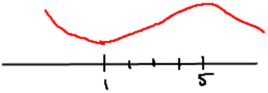
There will be no horizontal asymptotes because we can see that a positive value of x will cause the function to go to positive infinity and we know that there is symmetry with respect to the origin.

Decreasing/Increasing Intervals:

$$f'(x) = -3x^2 + 18x - 15$$
$$= -3(x^2 - 6x + 5)$$
$$= -3(x-5)(x-1)$$
$$x = 5, x = 1$$

	Test	Sign	f
$x<1$	$f'(0)$	$-$	$\searrow$
$1<x<5$	$f'(2)$	$+$	$\nearrow$
$x>5$	$f'(6)$	$-$	$\searrow$

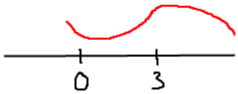
Local Min./Max.:
Using the values attained for increase/decrease, we can see that the min. value is at $(1, f(1)) = (1, -5)$ and the max. value is at $(5, f(5)) = (5, 27)$.



Concavity and Inflection Points:

$$f''(x) = -6x + 18$$
$$= -6(x-3)$$
$$x = 3$$

	f''	
$x<3$	$+$	$\cup$
$x>3$	$-$	$\cap$



Concavity and Inflection Points:
We know from the above graph that there is an inflection point at $\big(3, f(3)\big) = (3, 11)$.