

1. Use the guidelines of this section to sketch the curve.

$$y = x^3 + x$$

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**Domain:**  $(-\infty, \infty)$

**Intercepts:**

$$f(0) = (0)^3 + 0 = 0$$

$$0 = x^3 + x$$

$$0 = x(x^2 + 1)$$

$$x = 0$$

**Symmetry:**

Even?

Does  $f(x) = f(-x)$ ?

(Using 2 as test value.)

$$f(-2) = (-2)^3 + (-2) = -29$$

$$f(2) = (2)^3 + 2 = 10$$

So,  $f(x) \neq f(-x)$ .

Odd?

Does  $-f(x) = f(-x)$ ?

Using 2 as test value.

$$-f(x) = -(2^3 + 2) = -10$$

$$f(-2) = (-2)^3 - 2 = -10$$

So,  $-f(x) = f(-x)$ .

So, the function is odd.

### **Asymptotes:**

There will be no vertical asymptotes because there is no place where the function is undefined.

There will be no horizontal asymptotes because we can see that a positive value of  $x$  will cause the function to go to positive infinity and we know that there is symmetry with respect to the origin.

### **Decreasing/Increasing Intervals:**

$$f(x) = x^3 + x$$

$$f'(x) = 3x^2 + 1$$

This is always positive. So,  $f(x)$  is always increasing.

### **Local Min./Max.:**

Because  $f(x)$  is always increasing on its domain, there is no local max. or min.

### **Concavity and Inflection Points:**

$$f''(x) = 6x$$

$x < -\infty$ :  $f''(x)$  will be negative. So, for this interval,  $f(x)$  will be CD.

$x > \infty$ :  $f''(x)$  will be positive. So, for this interval,  $f(x)$  will be CU.

### **Concavity and Inflection Points:**

We know the  $x$  and  $y$  intercepts and the intervals of

concavity. If we graph a couple of points using the original equation, we can see the graph.

$$f(1) = (1)^3 + (1) = 2$$

$$f(2) = (2)^3 + (2) = 10$$

