21. Find the local maximum and minimum values of f using both the First and Second Derivative Tests.

$f(x)=x+\sqrt{1-x}$

Note: We need the radicand (in this case, $1-x$) to be 0 or greater. So, we set it equal to 0.
$1-x\geq 0 \quad \rightarrow \quad -x\geq -1 \quad \rightarrow \quad x\leq 1$.

So, the domain is [1, ∞).

We must keep this in mind as critical numbers can only come from the domain and critical numbers are what give us the local max and local min possibilities.

<u>First Derivative Test</u>

$f(x)=x+(1-x)^{\frac{1}{2}}$

$f'(x)=1+\frac{1}{2}(1-x)^{-\frac{1}{2}}\cdot(-1)$ (-1 from chain rule)

$= \quad 1-\frac{1}{2\sqrt{1-x}}$

Now, we must find any values of x for which this equation is undefined. It will be undefined at $x=1$. This value is in our domain and is a critical number, but because it is an endpoint on the domain, we can disregard it.

Now, we set the equation equal to 0.

$0= \quad 1-\frac{1}{2\sqrt{1-x}}$

$-1=-\frac{1}{2\sqrt{1-x}} \quad = \quad 1=\frac{1}{2\sqrt{1-x}} \quad \rightarrow$

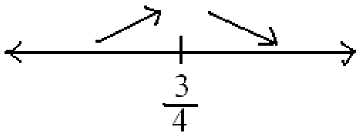
$2\sqrt{1-x}=1 \quad \rightarrow \quad \sqrt{1-x}=\frac{1}{2} \quad \rightarrow$

$(\sqrt{1-x})^2=(\frac{1}{2})^2 \quad \rightarrow \quad (1-x)=\frac{1}{4} \quad \rightarrow$

$-x=\frac{1}{4}-1 \quad \rightarrow \quad -x=\frac{1}{4}-\frac{4}{4} \quad \rightarrow \quad -x=-\frac{3}{4} \quad \rightarrow$

$x=\frac{3}{4}$

Interval	Sign of f'	f Inc. / Dec.
$(-\infty, \frac{3}{4})$	$f'(0)=\frac{1}{2}>0$	Inc.
$(\frac{3}{4}, 1)$	$f'(\frac{3}{8})<0$	Dec.



Local max. of $f(\frac{3}{4})=\frac{3}{4}+\sqrt{\frac{1}{4}}$

$= \frac{3}{4}+\frac{1}{2} = \frac{5}{4}$

<u>Second Derivative Test</u>

$$f'(x)=1-\frac{1}{2}(1-x)^{-\frac{1}{2}}$$

$$f''(x)=-\frac{1}{4}(1-x)^{-\frac{3}{2}}$$

$$f''(\frac{3}{4})=-\frac{1}{4}(1-\frac{3}{4})^{-\frac{3}{2}}$$

$$= -\frac{1}{4}(\frac{1}{4})^{-\frac{3}{2}}$$

$$= -\frac{1}{4}(4)^{\frac{3}{2}}=-4^{\frac{1}{2}}=-2<0$$