53. Find the absolute maximum and absolute minimum values of *f* on the given interval.

$f(x)=\frac{x}{x^2+1}$, [0, 2]

Note: this is a closed interval.

f is a continuous function. We know this because the only possible point of discontinuity is where the denominator $x^2+1=0$. But, there's no real number that can be squared, add one to, and get zero. Because any real number squared will be positive or zero. So, $x^2+1\neq 0$. So, we know that this function is defined everywhere and is continuous everywhere.

Because *f* is continuous on this closed interval, the Closed Interval method can be used.

First, the critical values must be found. These must be inside of the given interval.

To do this, first look for places in the domain that make the derivative zero or make the derivative not exist.

$$f'(x)=\frac{(x)'(x^2+1)-(x)(x^2+1)'}{(x^2+1)^2} =$$

$$\frac{(x^2+1)-(2x^2)}{(x^2+1)^2}$$

The only way for $f'(x)$ to be 0 is for the numerator to be equal to 0.

$$x^2+1-2x^2=0 \rightarrow -x^2=-1 \rightarrow x^2=1 \rightarrow$$

$$x=\pm 1$$

We can take out the negative as it is not in the given interval.

Now we check values in the original function *f*.

We know from the Closed Interval Method that the absolute maximum and absolute minimum have to occur either at one of the endpoints or at a critical number inside the interval.

$f(0)=0$

$f(1)=0.5$

$f(2)=0.4$