

37. Find the critical numbers of the function

$$h(t) = t^{\frac{3}{4}} - 2t^{\frac{1}{4}} .$$

Note that we are taking 4th roots and we can only take the even roots of positive numbers or zero.

So, Domain: $t \geq 0$ $[0, \infty)$

$$\begin{aligned} h'(t) &= \frac{3}{4}t^{-\frac{1}{4}} - \frac{1}{2}t^{-\frac{3}{4}} \\ &= \frac{3}{4\sqrt[4]{t}} - \frac{1}{2\sqrt[4]{t^3}} \end{aligned}$$

We can see that $h'(t)$ is undefined at $t=0$.

Now, solve $h'(t)=0$.

$$\frac{3}{4\sqrt[4]{t}} - \frac{1}{2\sqrt[4]{t^3}} = 0$$

$$\frac{3}{4\sqrt[4]{t}} = \frac{1}{2\sqrt[4]{t^3}}$$

$$\frac{6t^{\frac{3}{4}}}{t^4} = \frac{4t^{\frac{1}{4}}}{t^4}$$

$$6t^{\frac{1}{2}} = 4$$

$$(t^{\frac{1}{2}})^2 = \left(\frac{2}{3}\right)^2$$

$$t = \frac{4}{9}$$

So, the critical numbers are $t=0$, $\frac{4}{9}$.